College Algebra

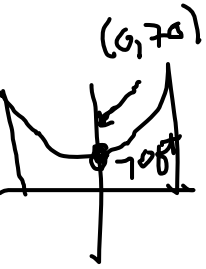
Word Problems Day

Golden Gate Bridge

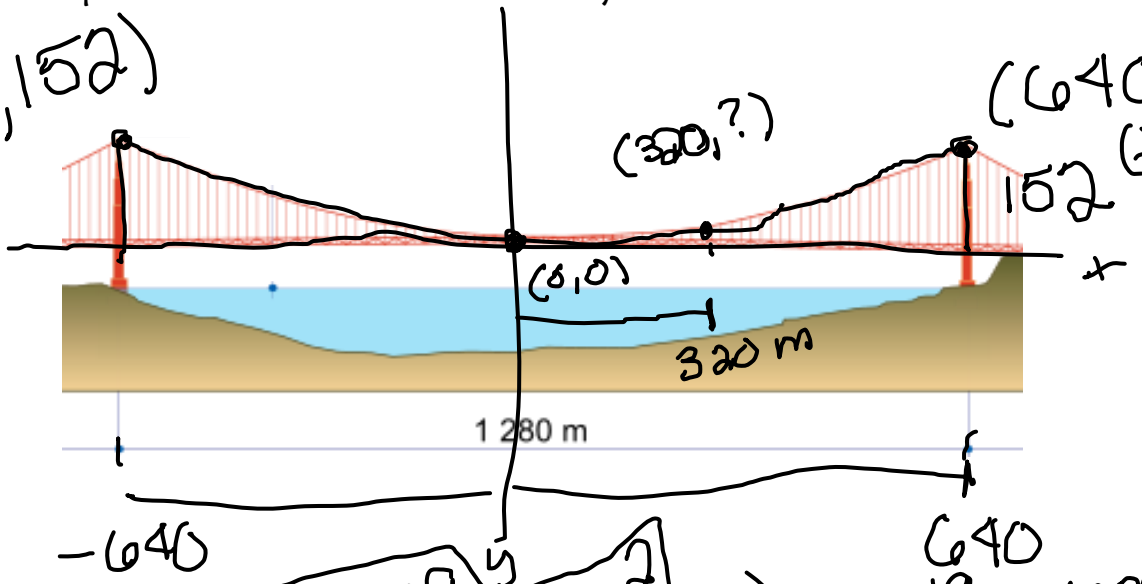
Each cable of the Golden Gate Bridge is suspended (in the shape of a parabola) between two towers that are 1280 meters apart. The top of each tower is 152 meters above the roadway. The cables touch the roadway at the midpoint between the towers.

a) Write an equation that models the cables of the bridge.

b) Find the height of the suspension cables over the roadway at a distance of 320 meters from the center of the bridge.



(-640, 152)



$y = a(x-h)^2 + k$

$y = a(x-0)^2 + 0$

$152 = a(640)^2$

$\frac{152}{409600} = \frac{409600a}{409600}$

$a = \frac{19}{51,200}$

a) $y = \frac{19}{51,200}x^2$

b) $y = \frac{19}{51200}(320)^2$

$y = 38$ meters

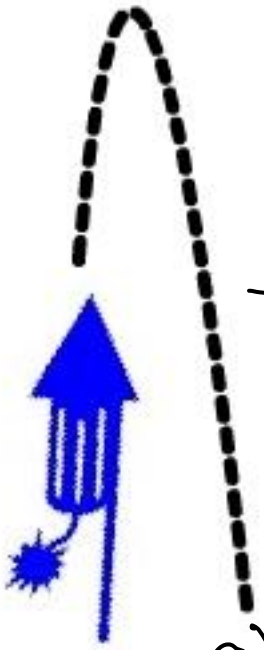
The height of the suspension cables over the roadway is 38m.

Projectile Motion:

A bottle rocket propelled straight upward from the ground reaches a maximum height of 784 feet above ground level after 7 seconds. Let the quadratic function $d(t)$ represent the distance above ground level (in feet) t seconds after the projectile is released.

a) Find $d(t)$

b) At what time will the bottle rocket be 640 feet off the ground?



vertex form $\leftarrow y = a(x-h)^2 + k$

$y = a(x-7)^2 + 784$

$0 = a(0-7)^2 + 784$

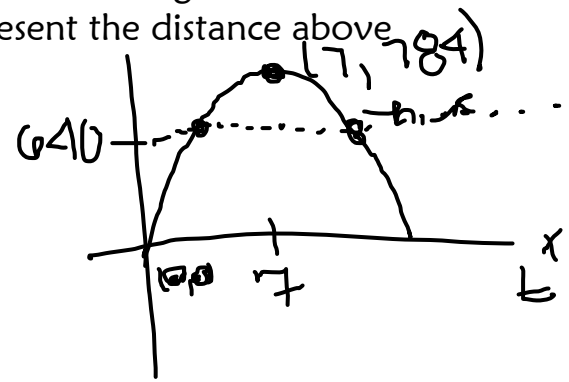
$0 = 49a + 784$

$-784 \quad -784$

$\frac{-784}{49} = \frac{49a}{49}$

$a = -16$

a) $y = -16(x-7)^2 + 784$



b) $y = h$

$640 = -16(x-7)^2 + 784$

$-784 \quad -784$

$\frac{-144}{-16} = \frac{-16(x-7)^2}{-16}$

$9 = (x-7)^2$

$\pm 3 = (x-7)$

$+7 \quad +7$

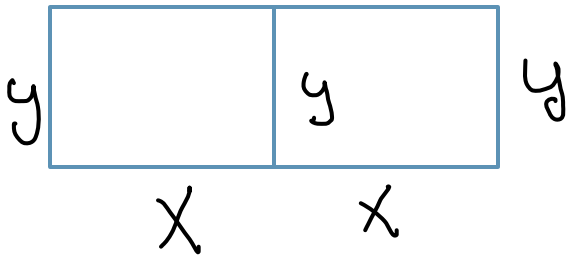
$x = 7 \pm 3$

$7 + 3 = 10$ sec

$7 - 3 = 4$ sec

Fencing

You have 200 meters of fencing to enclose two adjacent rectangular corrals. The total area of the enclosed region is 1400 square meters. What are the dimensions of each corral? (The corrals are the same size.)



$P = 4x + 3y$

$200 = 4x + 3y$

$-4x \quad -4x$

$\frac{200 - 4x}{3} = \frac{3y}{3}$

$\frac{200 - 4x}{3} = y$

$A = l \cdot w$

$A = 2x(y)$

$1400 = 2x\left(\frac{200 - 4x}{3}\right)3$

$4200 = 400x - 8x^2$

$-400x + 8x^2 \quad -400x + 8x^2$

$8x^2 - 400x + 4200 = 0$

$8(x^2 - 50x + 525) = 0$

$8(x - 35)(x - 15) = 0$

$x = 35 \quad x = 15$

1st $x = 35$

$4(35) + 3y = 200$

$y = 20$

$35\text{m} \times 20\text{m}$

or

2nd $x = 15$

$4(15) + 3y = 200$

$y = 46.\overline{66}$

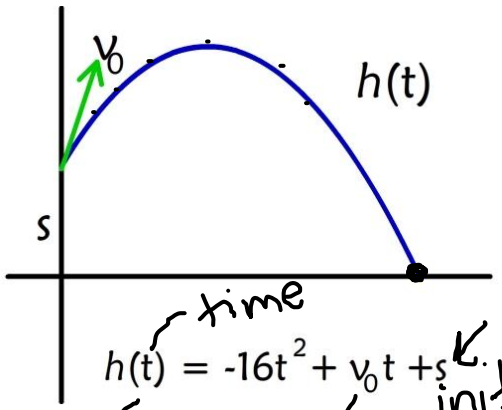
$= 46\frac{2}{3}$

$15\text{m} \times 46\frac{2}{3}\text{m}$

$46.\overline{66} = \boxed{46.67}$

Projectile Problem:

An object thrown from the ground at an initial speed of 32 feet per second. How long will it take for the object to come back to the ground?



$h(t) = -16t^2 + v_0t + s$

$h(t) = -16t^2 + 32t + 0$

$h(t) = -16t^2 + 32t$

$0 = -16t^2 + 32t$

$0 = -16t(t-2)$

$-16t = 0$

$-t = 0$

$t = 0$ sec

$t - 2 = 0$

$t = 2$ sec

Work Problem:

A water tower has two drainpipes attached to it. Working along, the smaller pipe would take 20 minutes longer than the larger pipe to empty the tower. If both drainpipes work together, the tower can be drained in 40 minutes. How long would it take the small pipe, working alone, to drain the tower? (Round your answer to the nearest tenth of a minute.)

Pipe	Time to Complete the Job Alone	Work Rate	Time Working	Portion of Job Completed
Smaller	t + 20 min.	$\frac{1}{t+20}$	× 40	$\frac{40}{t+20}$
Larger	t min.	$\frac{1}{t}$	× 40	$\frac{40}{t}$

work done smaller → $\frac{40}{t+20}$ + $\frac{40}{t}$ = 1 ← work done by large

1 complete job

$t(t+20)$

$$\left(\frac{40}{t+20}+\frac{40}{t}\right)=(1)^{t(t+20)}$$

Solve

$$t = 30 \pm 10\sqrt{17}$$

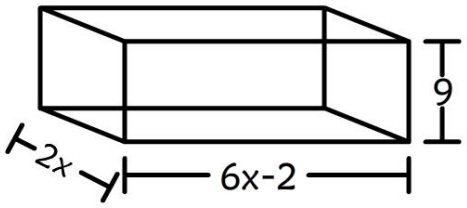
$$t = 30 + 10\sqrt{17}$$

$$= 71.23 \sim 91.2 \text{ minutes}$$

Omit the negative solution. The amount of time required by the small pipe is represented by $t + 20$, it would take approximately 20 + 71.2 or 91.2 minutes.

Geometry:

An open box with a rectangular base of 2x inches by 6x-2 inches has a height of 9 inches. The volume of the box is 1584 cubic inches. Find the dimensions of the box.



$V = l \cdot w \cdot h$

$1584 = (6x-2)(2x)(9)$

$1584 = (12x^2 - 4x)9$

$0 = 108x^2 - 36x - 1584$

$0 = 36(3x^2 - x - 44)$

$0 = 36(3x+11)(x-4)$

$3x+11 = 0$

$x = -\frac{11}{3}$

no neg dimension

$x-4=0$

$x=4$

$x=4$

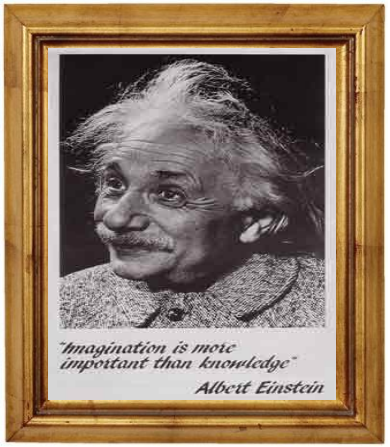
$2(4) = 8$

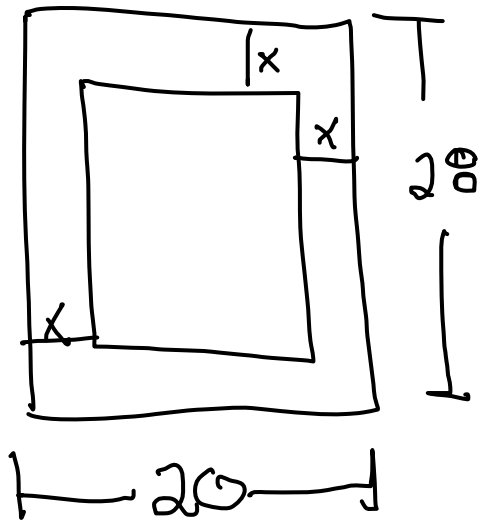
$6(4)-2 = 22$

8in x 22in x 9in

Design:

A rectangular picture frame of uniform width has outer dimensions of 28 inches by 20 inches. How wide (to the nearest tenth of an inch) must the frame be to display an area of 468 square inches?





$A = l \cdot w$

$468 = (20-2x)(28-2x)$

$4x^2 - 96x + 92 = 0$

$4(x^2 - 24x + 23) = 0$

$4(x-23)(x-1) = 0$

$x = 23$, $x = 1$

does not work

$(20 - 2(23))$

negative

The frame must be 1 inch to display an area of 468 in^2